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Keywords: Financial Crisis; Volatility; High Frequency Data; Realized GARCH.

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1 Introduction

The aim of this paper is to study financial volatility during the Global Financial Crisis, and analyze the days with the largest volatility shocks. We subsequently use high-frequency data to identify the exact timing of each shock, which gives us information about the events that likely caused the volatility shocks. Interestingly, the largest volatility shock is found to coincide with a technical problem in the trading system, while the days with large decline in volatility are mainly associated with government interventions.

The relationship between important financial/economic events and our realized measures of volatility is illustrated in Figure 1. The figure presents a daily annualized realized measure of volatility covering the period January 3, 1997 to December 31, 2009. The realized measures are computed from high-frequency prices of an exchange-traded fund (SPDR) that closely mimics the S&P 500 index. Several important clusters of volatility are observed and associated with major economic events that occurred during this period, including the Asian crisis, the Russian crisis, the Dot-com bubble burst, 9/11, and Lehman Brothers collapse. The highest measured value of volatility was recorded on October 10th, 2008, at 165.7 (annualized).

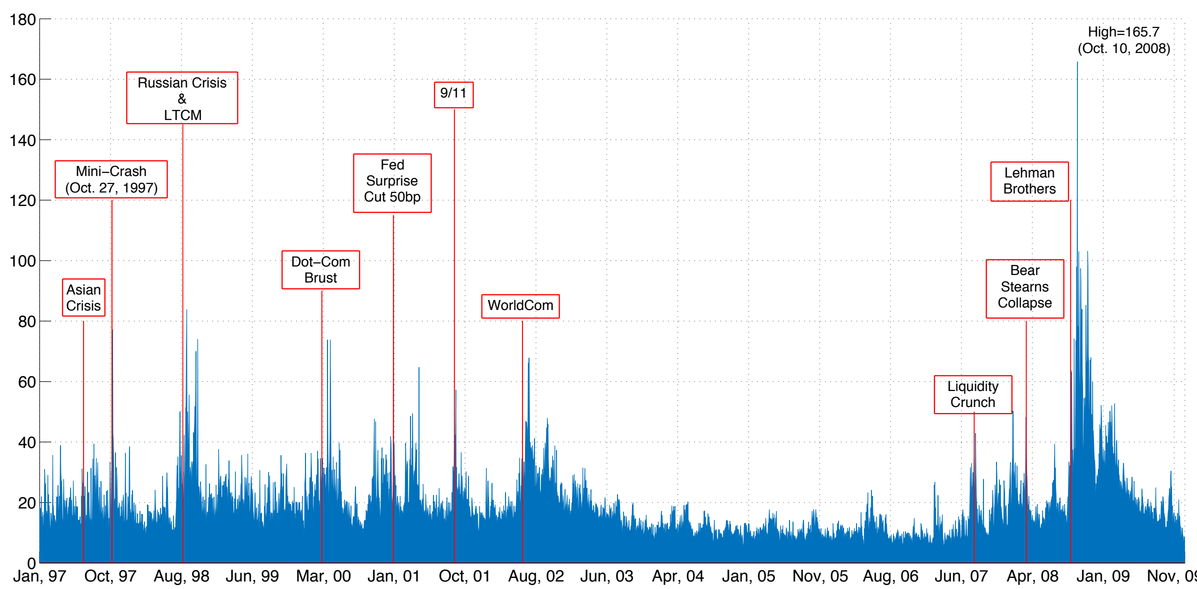

Figure 1: Annualized realized volatility for the period of 1997-2009 and some of the major crises and events.

In this paper we first utilize the recently developed Realized GARCH framework (Hansen et al., 2012) to extract daily volatilities. This framework uses accurate realized measures of volatility computed from high-frequency data, which facilitates a measure of daily volatility shocks. Because the Global Financial Crisis was an unusually volatile period, with several unusually large shocks, we propose a new variation of the Realized GARCH model which is less sensitive to outliers. This variant of the model improves the empirical fit during the crisis period, albeit the improvements are modest relative to those obtained with robustification of conventional GARCH models, see e.g. Harvey (2013, p. 13).

Knowledge of financial volatility has increased considerably over the last decade, revolving around two main lines of enquiry: measuring and modeling volatility. This is in part due to the increased availability of high-frequency financial price data, which has inspired the development of novel econometric methods that substantially improved the ex-post volatility measurement.

The impetus to the vastly growing literature on measuring volatility came largely from Andersen and Bollerslev (1998), who documented that the realized variance, computed as the sum of squared intraday returns, provides an accurate measurement of daily volatility. The stochastic properties of the realized variance were subsequently studied in Andersen et al. (2001), Barndorff-Nielsen and Shephard (2002), Meddahi (2002), Andersen et al. (2003), Mykland and Zhang (2009). In the meantime, a large number of improved proxies of volatility, which account for market microstructure noise, were introduced by Zhang et al. (2005), Barndorff-Nielsen et al. (2008), Hansen and Horel (2009), *inter alios.*

The improved measures of volatility motivated the development of volatility models that make use of realized measures. For instance, Engle and Gallo (2006) proposed the Multiplicative Error Model (MEM), which jointly models returns and realized measures of volatility via a multiple latent volatility process framework. The MEM framework was subsequently refined and used by Shephard and Sheppard (2010), who refer to their model as the HEAVY model. More recently, Hansen et al. (2012), see also Hansen and Huang (2016) and Hansen et al. (2014), introduced the Realized GARCH model that takes a different approach to the joint modeling of returns and realized volatility measures. The key characteristics of the Realized GARCH framework is the use of a measurement equation that ties the realized measure to the underlying conditional variance.

In this paper we propose and study a new variant of the Realized GARCH model that is sought to be robust to outliers. The new structure is inspired by Harvey (2013) who demonstrated that conventional GARCH models can be severely influenced by large returns with unfortunate empirical consequences. Harvey (2013) proceeded by proposing a score-driven model that can overcome the problem. Instead of having returns impact volatility directly, Harvey

(2013) use their corresponding score to model the dynamic properties of volatility, where the score is deduced from a t-distribution. The resulting structure effectively dampens outliers in an intuitive manner. Our robustified Realized GARCH borrows the outlier dampening feature of the score that was used in Harvey (2013).

The paper is organized as follows. Section 2 introduces the modeling framework including the robustified Realized GARCH specification. The empirical analysis is presented in Section 3. In Section 4 we discuss the news related to the largest volatility shocks. Section 5 concludes. Appendix A presents supporting theoretical results and Appendix B has additional empirical results.

2 Modeling Framework

2.1 Key Variables

We are to study volatility of asset returns, r_t. In the empirical analysis we use the exchange traded index fund, SPY, to define daily returns because it closely tracks the S&P 500 index returns and provides us with readily available high-frequency data. The conditional variance of daily returns is denoted by:

$$h_t = \mathrm{var}(r_t|\mathcal{F}_{t-1}), \tag{1}$$

where $\{\mathcal{F}_t\}$ is a filtration to which r_t is adapted. The volatility shock – the key variable in this analysis – is defined by:

$$v_t = \mathbb{E}(\log h_{t+1}|\mathcal{F}_t) - \mathbb{E}(\log h_{t+1}|\mathcal{F}_{t-1}), \tag{2}$$

so that $100 \times v_t$ is the percentage shock to volatility, induced by news on the t^{th} day.

In the rest of this section we detail the econometric modeling of returns and realized measures of volatility, which will lead to our empirical estimates of volatility shocks. After introducing the Realized GARCH framework we detail the robustified version of the model that we introduce in this paper. Readers who are primarily interested in the empirical analysis and less interested in the details of the econometric models can skip the rest of this section and go directly to the empirical analysis in Section 3.

2.2 Realized GARCH Framework

The Realized EGARCH model of Hansen and Huang (2016) (with a single realized measure of volatility) is given by the following three equations:

$$
\begin{aligned}
r_t &= \mu + \sqrt{h_t} z_t, & (3) \\
\log h_t &= \omega + \beta \log h_{t-1} + \tau(z_{t-1}) + \gamma u_{t-1}, & (4) \\
\log x_t &= \xi + \varphi \log h_t + \delta(z_t) + u_t, & (5)
\end{aligned}
$$

where $\tau(z) = \tau_1 z + \tau_2(z^2 - 1)$ and $\delta(z) = \delta_1 z + \delta_2(z^2 - 1)$. Here, z_t and u_t are typically assumed to be mutually and serially independent and modeled with the specifications: $z_t \sim \text{iid}(0, 1)$ and $u_t \sim \text{iid}(0, \sigma_u^2)$.

The three equations are labelled as the *return equation,* the *GARCH equation*, and the *measurement equation,* respectively. The first two form the basis for a GARCH-X model, similar to that estimated by Engle (2002), Barndorff-Nielsen and Shephard (2007), and Visser (2011). The measurement equation is a key characteristic of the Realized GARCH framework, which ties the (ex-post) realized measure, x_t, to the latent (ex-ante) conditional variance, h_t. A GARCH-X model is – in isolation – an incomplete description of the data, because it does not specify a model for the realized measure. A complete specification of the dynamic properties of both returns and realized measures is achieved by means of the measurement equation. An alternative approach to completing the GARCH-X model, which involves additional latent variables, was proposed by Engle and Gallo (2006), see also Shephard and Sheppard (2010).

Some of the key features of this model are captured by β, which measures the persistence of volatility, and by $\tau(z_{t-1}) + \gamma u_{t-1}$, which estimates the innovation in the conditional volatility. For instance, γu_{t-1} captures the impact that the realized measure has on the next period conditional variance. The functions $\tau(z)$ and $\delta(z)$ are called the leverage functions, as they specify a dependence between returns and volatility commonly referred to as the *leverage effect.* Hansen et al. (2012) explored different leverage functions and found a simple quadratic form to be satisfactory in practice. We adopt the same structure in our estimation. In addition, the term $\tau(z)$ makes reference to the *news impact curve* introduced by Engle and Ng (1993), which shows how positive and negative returns impact expected future volatility.

2.3 Robustified Realized GARCH

Several unusually large shocks to returns and volatility occurred during the Global Financial Crisis. Large shocks pose challenges to conventional GARCH models, as they are highly sensitive to large returns. This motivated Harvey (2013) to suggest a more robust dynamic structure that utilizes the conditional scores of the model. This type of model is known as the dynamic conditional score (DCS) or generalized autoregressive score (GAS) model, see Harvey (2013) and Creal et al. (2012, 2013), respectively.

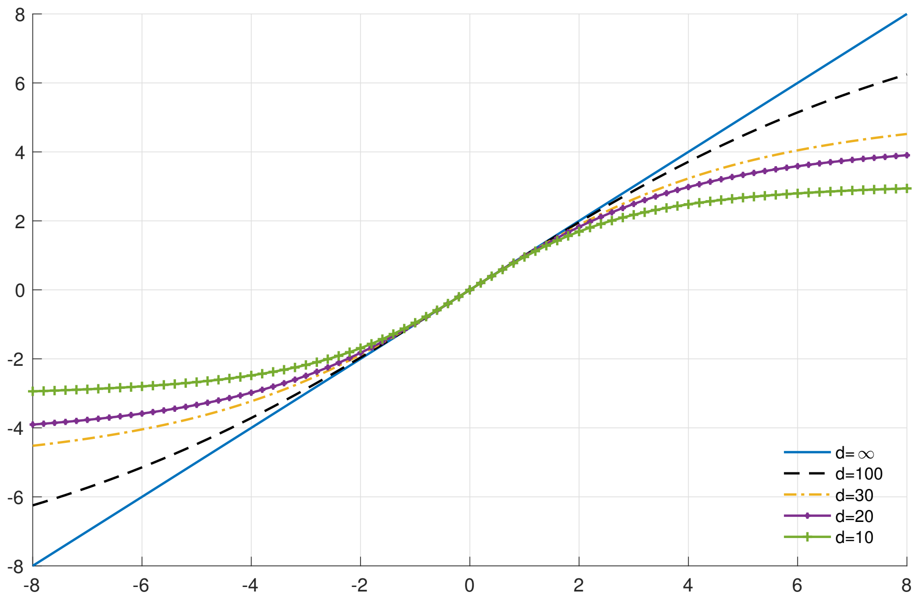

Figure 2: The transformation $x \mapsto x/\sqrt{1+x^2/d}$ for various values of d.

We adopt some insights from Harvey (2013) by introducing parameters that serve to dampen the impact of outliers in returns. We achieve this by substituting z_t with $\tilde{z}_t = z_t/\sqrt{1+z_t^2/d_z}$ in the GARCH equation, where d_z is a parameter to be estimated. The transformation is illustrated in Figure 2 for different values of d. Harvey (2013) deduced the transformation from the score function within a conventional GARCH model, where a univariate time-series of returns is being modeled, see Appendix A.1 for details. In the present context we are modeling both returns and realized measures and both might be affected by outliers (*i.e.*, outliers to returns and outliers in the realized measures, which would translate into unusually large values for z_t and u_t, respectively). Therefore, we adopt a similar adjustment of u_t, which measures the shocks to volatility, and substitute $\tilde{u}_t = u_t/\sqrt{1+\left(u_t/\sigma_u\right)^2/d_u}$ for u_t in the GARCH equation. Here, d_u is a second robustness parameter to be estimated, analogous to d_z, and we note that the standard Realized GARCH model emerges in the limit as $d_z, d_u \to \infty$.

The robustified Realized GARCH model has the following structure:

$$r_t = \mu + \sqrt{h_t} z_t, \tag{6}$$

$$\log h_t = \omega + \beta \log h_{t-1} + \tau(\tilde{z}_{t-1}) + \gamma \tilde{u}_{t-1} \tag{7}$$

$$\log x_t = \xi + \varphi \log h_t + \delta(z_t) + u_t, \tag{8}$$

where $\tilde{z}_t = z_t / \sqrt{1 + z_t^2 / d_z}$ and $\tilde{u}_t = u_t / \sqrt{1 + \left(u_t^2/\sigma_u^2\right)/d_u}$, with the leverage functions given by $\tau(\tilde{z}) = \tau_1 \tilde{z} + \tau_2(\tilde{z}^2 - 1)$ and $\delta(z) = \delta_1 z + \delta_2(z^2 - 1)$. Additional variants of the robust model are estimated and compared, see Appendix B for details. In our quasi maximum likelihood estimation we model z_t and u_t to be mutually and serially independent, with $z_t \sim \mathrm{iid}(0,1)$ and $u_t \sim \mathrm{iid}(0, \sigma_u^2)$.

Within the model defined by (6)-(8), the volatility shock which was defined in (2), v_t, is in the present model given by:

$$v_t = \tau(\tilde{z}_t) + \gamma \tilde{u}_t. \tag{9}$$

Therefore, the volatility shock has two components. The first component is the news impact curve that is well known from conventional GARCH models. The second term captures the additional information about future volatility that is embodied in the realized measure of volatility. This term illustrates another advantage of using realized measures, as an improved measurement of the volatility shock is made available within the Realized GARCH framework.

3 Empirical Analysis

3.1 Data Description

We use high-frequency prices for the exchange traded fund, SPY, which closely tracks the S&P 500 index. The high-frequency data are obtained from the Trade and Quote (TAQ) database, and our full sample spans the period from January 1, 1997 to December 31, 2009.

We follow the standard practice in the GARCH literature and model daily close-to-close returns. The realized measure of volatility is an estimate of volatility over the part of the day where high-frequency data is available, typically from 9:30 am to 4:00 pm, which is obviously less than close-to-close volatility that is relevant for daily returns. Hansen et al. (2012) found that about 75% of volatility occurs during the 6.5 hours with active trading, and estimated φ to be very close to one, which suggests that the realized measure is proportional to daily volatility. As our realized measure of volatility, x_t, we adopt the realized kernel (RK) by Barndorff-Nielsen et al. (2008). To this end we use the Parzen kernel function and a bandwidth that ensures robustness to market microstructure noise, using the implementation in Barndorff-Nielsen et al. (2011), which guarantees a positive estimate. The positivity is useful because we will be specifying our model for the logarithmically transformed volatility. Prior to computing intraday returns and realized measures, we preprocess the high-frequency data using the cleaning procedures of Barndorff-Nielsen et al. (2009). We also remove unusually quiet trading days (such as days with limited trading hours) around Thanksgiving and Christmas in order to avoid obvious outliers in the realized measures.

In order to quantify the volatilities using an intuitive scale, we will typically report the conditional variance and realized measure at an annualized volatility scale. The annualized realized volatility is defined from the realized kernel estimates by:

$$\mathrm{Rvol}_t = \sqrt{250 \times \hat{c} \times \mathrm{RK}_t}, \qquad \hat{c} = \frac{\sum_t r_t^2}{\sum_t \mathrm{RK}_t}, \tag{10}$$

while the annualized conditional variance (volatility) is defined by $\mathrm{Cvol}_t = \sqrt{250 \times h_t}$. The constant $\hat{c}$ adjusts for the fact that RK_t measures volatility over the part of the day that high-frequency data are available, and not the whole day. The adjustment is $\hat{c} \simeq \frac{4}{3}$ because about 75% of daily volatility occurs during the hours between 9:30 am and 4:00 pm.

3.2 Estimation Results

When modeling returns with conventional GARCH models, the specification of the conditional mean typically does not make much difference. This is also true within the Realized GARCH framework. In the present application we have estimated models with constant μ as well as models where μ is set to zero. The unrestricted estimate of μ is small and insignificant, and the resulting time series for $\hat{h}_t$ are virtually identical whether μ is estimated or simply set to zero. The empirical results reported in this paper are for models where we have imposed the constraint $\mu = 0$.

Next we present estimation results for the robustified Realized GARCH model for the period of January 1, 2006 to December 31, 2009. The numbers in brackets are robust standard errors.[1] We have also estimated the same specification for the full sample period, January 3, 1997 to December 31, 2009, which results in very similar point estimates. These results are presented

[1] Robust standard errors are computed using the sandwich estimator, see Bollerslev and Wooldridge (1992).

in Appendix B.

$$
\begin{aligned}
r_t &= \sqrt{h_t} z_t, \\
\log h_t &= \underset{(0.010)}{0.017} + \underset{(0.006)}{0.969} \log h_{t-1} + \underset{(0.061)}{0.407} \tilde{u}_{t-1} - \underset{(0.017)}{0.180} \tilde{z}_{t-1} + \underset{(0.014)}{0.056} (\tilde{z}_{t-1}^2 - 1), \\
\log x_t &= \underset{(0.082)}{-0.532} + \underset{(0.070)}{1.019} \log h_t - \underset{(0.017)}{0.132} z_t + \underset{(0.009)}{0.036} (z_t^2 - 1) + u_t,
\end{aligned}
$$

with $\hat{\sigma}_u^2 = \underset{(0.008)}{0.154}, \hat{d}_z = 27.689, \hat{d}_u = 5.904$; we do not have standard errors for d_z and d_u, but approximate confidence sets can be obtained by inverting the likelihood ratio statistics.

All key parameters are statistically significant and their signs are meaningful. For instance, the value of the coefficient for $\tilde{u}_{t-1}$ is $\hat{\gamma} = 0.407$, which shows that the realized measure provides an informative signal about future volatility, $\hat{\beta} = 0.969$ reflects the high persistence in volatility, and $\hat{\varphi} = 1.019$ suggests that the realized measure is proportional to the conditional variance. The implication is that a fixed proportion of daily volatility occurs during the 6.5 hours that the market is open.

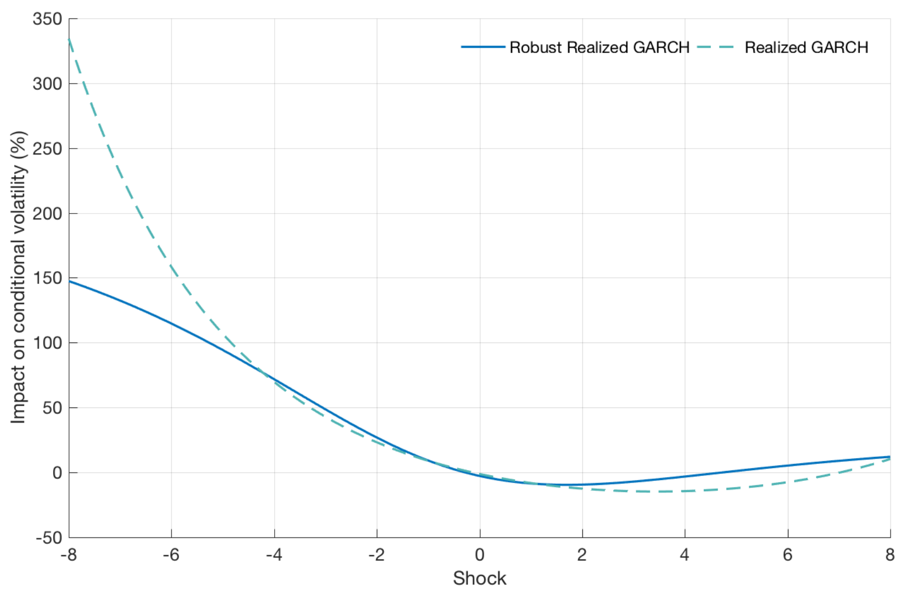

Figure 3: The estimated News Impact Curves based on the Realized GARCH model (dashed) and the robustified Realized GARCH model (solid).

The asymmetric response in volatility to return shocks (leverage effect) is encapsulated in $\hat{\tau}_1 = -0.180$ and $\hat{\delta}_1 = -0.132$. The estimated response in volatility to studentized return shocks, z_t, is summarized by the news impact curve. The news impact curve is displayed in Figure 3, for both the robustified Realized GARCH model and the Realized GARCH model.

The asymmetric response is pronounced in both models, with negative return shocks having a disproportionally larger impact on volatility than positive return shock of the same magnitude. Figure 3 highlights differences between the robust and non-robust Realized GARCH models, specifically that the former dampens the impact on volatility on days with extreme negative returns shocks.

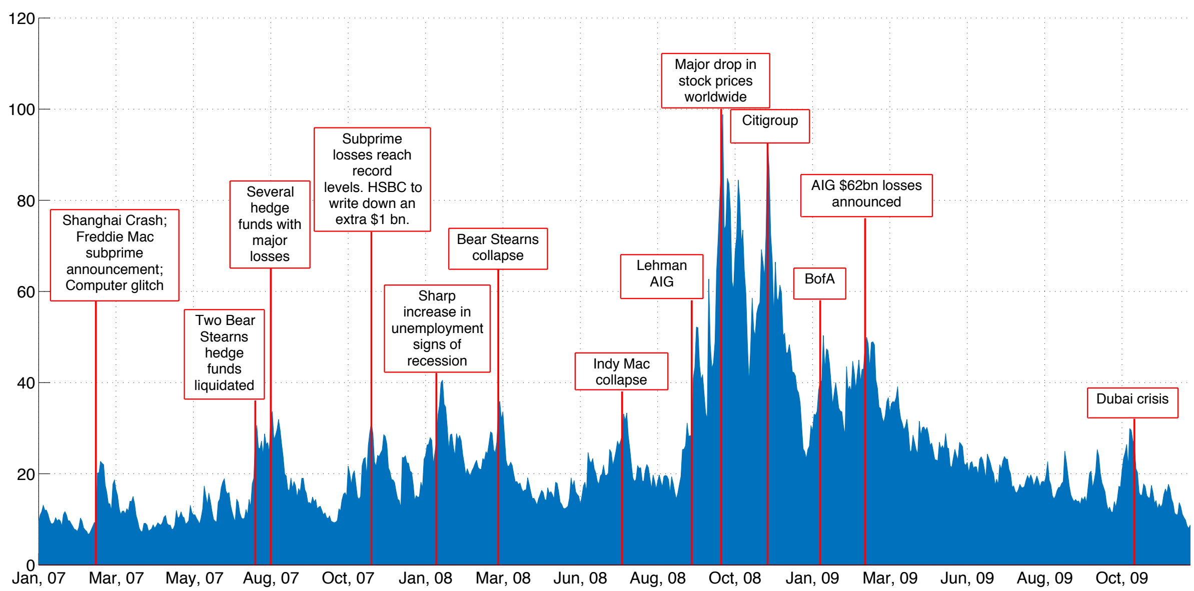

Figure 4: The conditional variance (annualized volatility) estimated with the robustified Realized GARCH model, along with makers of several major events.

The time series of the conditional variance, h_t, implied by the estimated model is presented in Figure 4 along with markers of some of the main events during the Global Financial Crisis. The first spike in volatility was on February 27, 2007, and several other spikes in volatility are associated with key events such as those related to Bears Stearns, the collapse of Lehman Brothers, and the unexpected down-vote of the $700 billion banking-rescue package by the House of Representatives, etc. We will undertake a closer investigation of the largest volatility spikes in the next section of the paper.

The volatility shock, $v_t = \mathbb{E}(\log h_{t+1}|\mathcal{F}_t) - \mathbb{E}(\log h_{t+1}|\mathcal{F}_{t-1}) = \tau(\tilde{z}_t) + \gamma\tilde{u}_t$, summarizes the effect that news on day t has on expected future volatility. It can be deduced from the estimated model using (9), and our estimates of v_t are presented in Figure 5 along with daily returns. As it turns out, the largest estimated volatility shock fell on February 27, 2007. This is partly due to the fact that volatility was relatively low prior to this date (about 9% annualized) so that a 118% increase in expected annualized volatility (which is what $v_t = 1.558$ translates into) did not bring the volatility to a record high level, but it was nevertheless the largest shock in

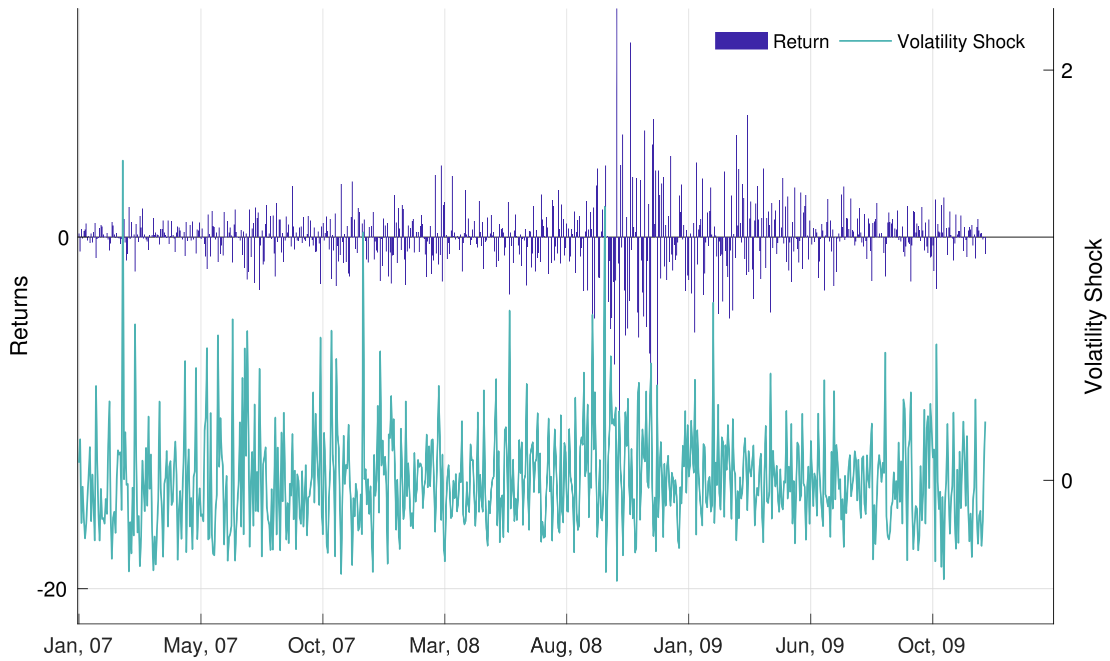

Figure 5: Returns, r_t, and volatility shocks, v_t.

percentage terms. The non-robust specification has $v_t = 2.295$ on February 27, 2007, which translates into a 215% increase in annualized volatility. (The values of v_t for both the robust model and non-robust model are given in Table B.2).

In Figure 6 we compare the non-robust Realized GARCH model with the new specification. The upper left panel displays the two series of h_t along with the realized measure of volatility (using an annualized scale). The two series of h_t are very similar, occasionally one can see the volatility of the non-robust specification spiking up a bit higher than that of the robust specification. The other three panels display the same series over three-week intervals that include the three largest volatility shocks in our sample. Large discrepancies between the volatility series are observed in the upper right panel following the event on February 27, 2007.

In response to the large realized measure of volatility and the negative return on February 27, 2007, we observe that the Realized GARCH reacts strongly to the large realization of returns and realized measure of volatility. The non-robust model predicts volatility to be much higher than what is actually observed in the realized measure the following day. The robust model performs better following this event, except for the second day, March 1st. About a week later, the two specifications produce very similar values for the conditional variance. Generally, we observe that the standard and the robust versions of the Realized GARCH model yield similar values for the conditional variance. This includes the periods around the second and the third largest volatility shocks, where the only noticeable difference is the day following the second

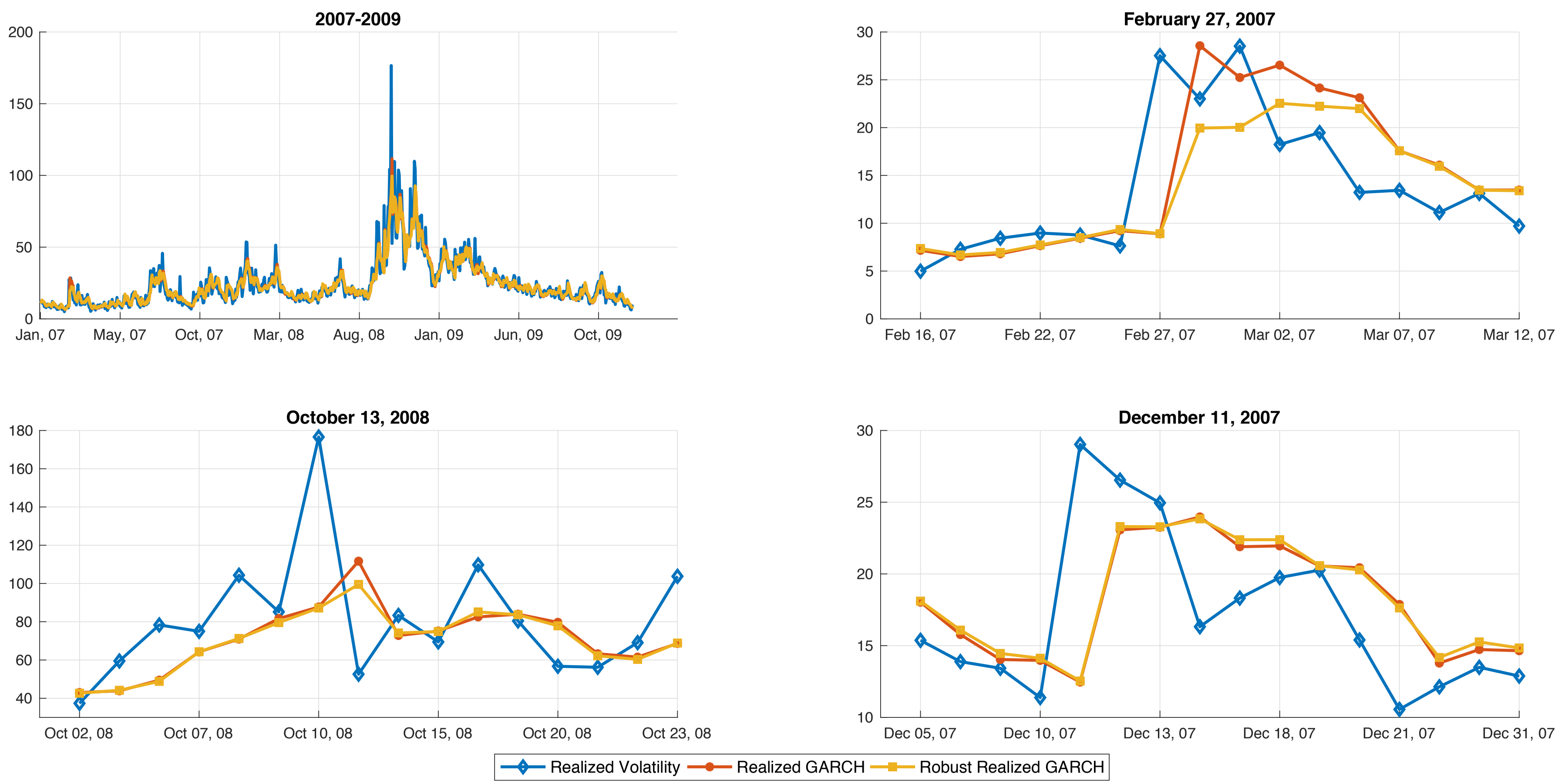

Figure 6: Evolution of the realized measure of volatility and the conditional volatility

Note: This figure presents the evolution of the realized measure of volatility and the conditional volatility for the period 2007-2009 and three sub-periods that surround the dates with the three largest volatility shocks: February 27, 2007, September 29, 2008, and December 11, 2007, respectively. The blue line represents the realized measure of volatility, while the red and green lines correspond to the estimated conditional volatility from the Realized GARCH and the robustified Realized GARCH models, respectively.

largest volatility shock.

Outliers in daily returns strongly influence the volatility in conventional GARCH models. The impact of daily returns is greatly reduced in Realized GARCH models that largely rely on the information provided by realized volatility measures. This highlights other advantages of using realized measures for the modeling of volatility. While robustification is important for conventional GARCH models, it is less important for Realized GARCH models in the present empirical analysis. Naturally, the robustification could be more important in other applications where large outliers are more prevalent.

In the next section we will focus on the dates with the largest volatility shocks.

4 News Related to the Largest Volatility Shocks

In this section we undertake a more detailed study of the days in our sample that we have associated with the largest volatility shocks. The positive (upwards) shocks are typically larger than the negative (downwards) shocks in volatility, both in terms of absolute changes and in percentages changes. Using the volatility shocks from the estimated robustified Realized GARCH model, we zoom in on the ten largest upwards shocks, which are listed in Table 1, and the five largest downwards volatility shocks that are listed in Table 2.

Table 1 lists the ten days with the largest positive volatility shocks along with the percentage changes in the S&P 500 and a list of selected news stories. Similarly, Table 2 lists the five dates with the largest percentage reduction in expected volatility. Volatility is often reported using an annualized volatility scale, such as $\sqrt{250h_t}$. The percentages volatility shock to $\sqrt{250h_t}$ is approximately given by $100(e^{\frac{1}{2}v_t} - 1)$, see Appendix A.2. For the positive volatility shocks this results in shocks that range from 44% to 118%, and the five downwards shocks range from -20% to -22%. It is interesting to note that all of the ten upwards volatility shocks are associated with large negative returns, whereas the five downwards volatility shocks all coincided with relatively large positive returns.

Table 1: Dates with the ten largest upwards volatility shocks and some key news

Date	**Vol. shock**	r_t	**News**
20070227	118%	-3.5%	[1] China stock market dropped by 8.8%. [2] Freddie Mac announced tightening standards on subprime loans. [3] NYSE trading interrupted because of a computer glitch around 3:00 pm. [4] News of a suicide bombing at the entrance to the main U.S. military base in Afghanistan during a visit by Dick Cheney, and pessimistic news on the U.S. economic growth.
20080929	95%	-8.8%	[5] The House of Representatives rejected the $700 billion banking-rescue package. [6] Wachovia announced the selling of the banking operation to Citibank. [7] The crisis spread to the European financial system (*e.g.*, the Icelandic government nationalizes the bank Glitnir).
20071211	83%	-2.5%	[8] Fed cut the federal funds rate by 0.25% to 4.25%. [9] Large subprime losses announced by Freddie Mac.
20090210	54%	-4.9%	[10] Obama administration unveiled a new rescue package, which was generally received with concerns that it would be inadequate. [11] Large layoffs announced by several companies, including General Motors, Wal-Mart Stores, UBS.
20080606	51%	-3.1%	[12] Unexpected large increase in May, 2008 unemployment rate announced (5.5% up from 5.0% in previous month). [13] Bond guarantors, MBIA and Ambac, were downgraded two notches from AAA to AA. [14] Lehman Brothers announced plans to raise $5-6 billion in fresh capital as it disclosed a large second-quarter loss.
20080915	50%	-4.7%	[15] Lehman Brothers Holdings Inc filed for bankruptcy protection. [16] Merrill Lynch acquired by Bank of America.
20070710	48%	-1.4%	[17] Standard and Poor's Rating Services added 612 securities to the "CreditWatch negative" list, because of high delinquency and foreclosure rates. Moody's Investors Service downgraded 399 securities and placed an additional 32 securities on review for possible downgrade.
20070313	46%	-2.0%	[18] Media reported concern about subprime lending. [19] The US dollar tumbled versus other major currencies.
20071101	44%	-2.6%	[20] Downgrade of Citigroup. [21] Credit Suisse reported a 31 percent drop in profits. [22] Exxon Mobil reported a bigger-than-expected drop in quarterly earnings. [23] Moody's, Standard & Poor's and Fitch put an estimated $70 billion worth of collateralized debt obligations on review for downgrading. [24] Economic reports on personal income and spending, manufacturing, foreclosure filings.
20070726	44%	-2.3%	[25] Wells Fargo & Co. announced that it will stop making subprime mortgages through brokers amid escalating late payments and defaults. [26] NYSE invoked trading curbs to slow trading due to the large price changes. [27] Homebuilders posted huge losses (new house sales tumbled 6.6%).

Note: Volatility shocks, returns on the S&P 500 index (source Yahoo Finance), and key events/news. Specific events and news stories related to the numbers 1,...,27 are collected in an extensive web appendix, see Banulescu et al. (2017).

For twelve of these days in the sample (those with the seven largest positive volatility shocks, and five largest negative volatility shocks) we present intraday high-frequency price data along with 13 realized measures of volatility, each computed over 30-minute intervals. Here we rely on the simpler realized variance, which is the sum of squared 1-minute returns. So, the volatility estimate for each of the 30-minute intervals is computed from 30 intraday returns. The realized variances are converted into an annualized volatility scale, by $\mathrm{RV} \mapsto \sqrt{250 \times 13 \times \hat{c} \times \mathrm{RV}}$ where $\hat{c} = \sum r_t^2 / \sum x_t$ is the constant defined in (10) that adjusts for the fact that the realized measures only compute volatility over a fraction of the day. For each of the twelve days we summarize some of the main news and use the high-frequency data to identify the key pieces of news, to the extent this is possible.

Table 2: Dates with the five largest downwards volatility shocks and selected news.

Date	**Vol. shock**	r_t	News
20081013	-22%	11.6%	[1] Governments to rescue banks through direct capital injections. [2] The European Central Bank attempts to revive credit market by making unlimited euro funds available. [3] The U.S. central bank to provide unlimited dollars to the European Central Bank, Bank of England and Swiss National Bank, allowing them to relieve pressures on commercial banks across their regions.
20091109	-21%	2.2%	[4] Finance ministers of the G-20 met over the weekend and pledged to keep the economic stimulus in place.
20071113	-20%	2.9%	[5] Positive statements from CEOs of Goldman Sachs and JP Morgan. [6] Wal-Mart Stores, Inc., reported higher than expected third-quarter earnings. [7] Oil price retreated from near high record levels. [8] Home sales index (for September, 2007) released in the afternoon. PHSI up 0.2% beating expectations of -2.5%.
20080930	-20%	5.4%	[9] Decline in volatility mainly due to the spike in volatility on the preceding day, that resulted from Congress's rejection of the banking-rescue package.
20071221	-20%	1.7%	[10] The Federal Reserve announced it had lent $20 billion to banks in order to support the credit markets. [11] The "Super SIV" rescue fund was canceled as the consortium claimed that "[it] is not needed at this time". [12] Encouraging economic news about personal income and spending.

Note: Volatility shocks, returns on the S&P 500 index (source Yahoo Finance), and key events/news. Specific events and news stories related to the numbers 1,...,12 are collected in an extensive web appendix, see Banulescu et al. (2017).

Tuesday, February 27, 2007 (+118%)

February 27, 2007 corresponds to the largest volatility shock in our sample, with a volatility shock $v_t = 1.558$ that translates into an expected 118% increase in volatility. On this day, the Dow Jones Industrial Average fell 416.02 points, which was the largest drop since 9/11, and the

S&P 500 and Nasdaq fell by about 3.5% and 3.9%, respectively.

There were several potentially distressing news stories by the time the (US) markets opened. The Chinese stock market had crashes, there were pessimistic news on the U.S. economic growth (e.g., on Monday, the Federal Reserve Chairman Alan Greenspan announced a potential fall of the economy into a recession by the end of 2007; report on the decline in the durable goods orders in January and on housing prices, etc.), and the U.S. military base in Afghanistan, which Vice President Dick Cheney was visiting, was attacked by a suicide bomber. Moreover, Freddie Mac announced tighter standards on subprime loans.

The subprime related news story from Freddie Mac is unlikely to have been of major significance to the market turmoil, because the tighter standards were only to be put into effect starting September 1, 2007. The Chinese crash is more likely to have been a contributing factor, as the Shanghai Composite Index had fallen -8.5%, allegedly caused by fears of new regulatory measures, such as possible trading taxes. However, this explanation also seems implausible when we turn to the evidence offered by high-frequency data.

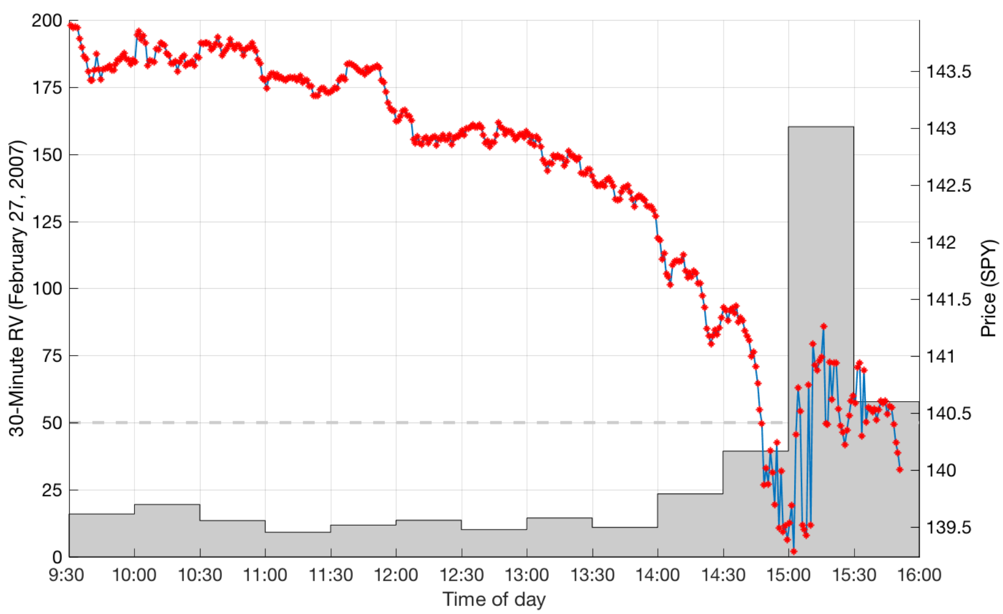

Figure 7: Intraday prices and the realized measure of volatility – February 27, 2007

Figure 7 presents the high-frequency prices (minute-by-minute) on the SPY along with realized variances computed over 30 minute intervals. It is evident that markets were not particularly disturbed by any of these news stories, including the Chinese crash. What stands out on this day is the increased price fluctuations that begin shortly before 15:00, causing volatility to jump by a factor of eight over a short period of time. This timing coincides with a computer glitch in the trading system. The glitch caused some trades not to be reported immediately, such that posted prices became stale. According to the Dow Jones spokeswoman: "around 2:00 pm [on that day] the market's extraordinary heavy trading volume caused a delay in the Dow Jones data systems. [...] and as we identified the problem we decided to switch to a back-up system and the result was a rapid catch-up in the published value of the Dow Jones Industrial Average." The back-up system was activated around 3:00 pm and at 3:02 pm the index fell by 160 points and continued its depreciation throughout the afternoon. The Dow Jones Industrial Average index fell by 546 points in the afternoon. The data for this day provides an excellent example of the valuable information that high-frequency data can offer, and shows that high-frequency data are essential for correctly pinpointing the news events that were the main sources for the market turmoil.

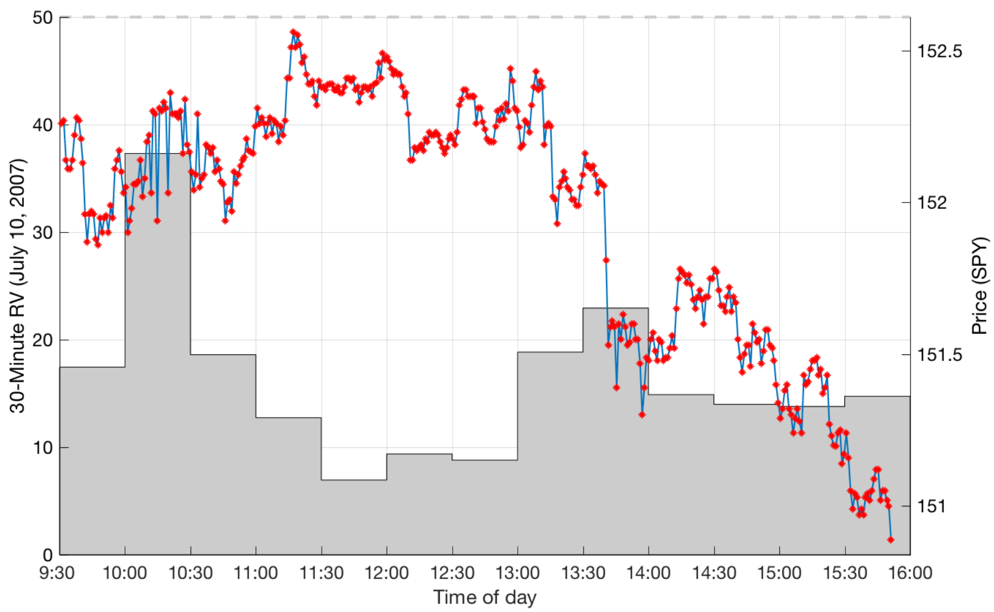

Figure 8: Intraday prices and the realized measure of volatility – July 10, 2007

Tuesday, July 10, 2007 (+48%)

On 10 July 2007 the rating agencies cut the rating for several subprime bonds. Standard and Poor's placed 612 securities backed by subprime mortgages on "CreditWatch negative". These 612 securities made up about 2 percent of all residential mortgage-backed securities in the US. Delinquencies exceeded historical norms by a wide margin and occurred at higher rates than the agency previously expected. This directly affected Bear Sterns, Citigroup, JP Morgan, Merrill

Lynch, and Morgan Stanley, which held a large amount of these securities in their portfolio. The same day, Moody's downgraded 399 securities and placed additional 32 on review for possible downgrade.

It was evident that these downgrades could have significant implications for the housing market, because borrowers with subprime adjustable rate mortgage (ARM) loans, would face difficulties refinancing their loans at an increased interest rates. Stricter underwriting standards made even more difficult for borrowers to refinance out of unaffordable ARMs, and the falling prices in the housing market placed an increasing number of borrowers "under water".

Standard and Poor's cited findings by Mortgage Asset Research Institute (MARI) as one of the reasons for the downgrades. MARI had reported a high incidence of fraud in loan applications, such as false or unsubstantiated claims about income, assets, and employment. Affected loans were known as "liar loans".

Tuesday, November 13, 2007 (-20%)

On November 13, 2007 the Dow rose by about 320 points. Goldman Sachs and JP Morgan were up 8.5% and 6.2%, respectively, after Goldman Sachs CEO, Lloyd Blankfein, said that the company would not suffer further significant losses related to subprime mortgages, and JP Morgan CEO, Jamie Dimon, downplayed its exposure to subprime debt. Other good news included Wal-Mart reporting higher than expected third-quarter earnings along with a positive outlook, and oil prices fell (U.S. light crude oil for December delivery fell by $3.45).

Another, significant news story was a 0.2% increase in the US Pending Home Sales (September, 2007), which was substantially better than the forecast of -2.5% and the -6.5% decline in US Pending Home Sales for the previous month. The release of this story coincides with the afternoon rally in the market on this date.

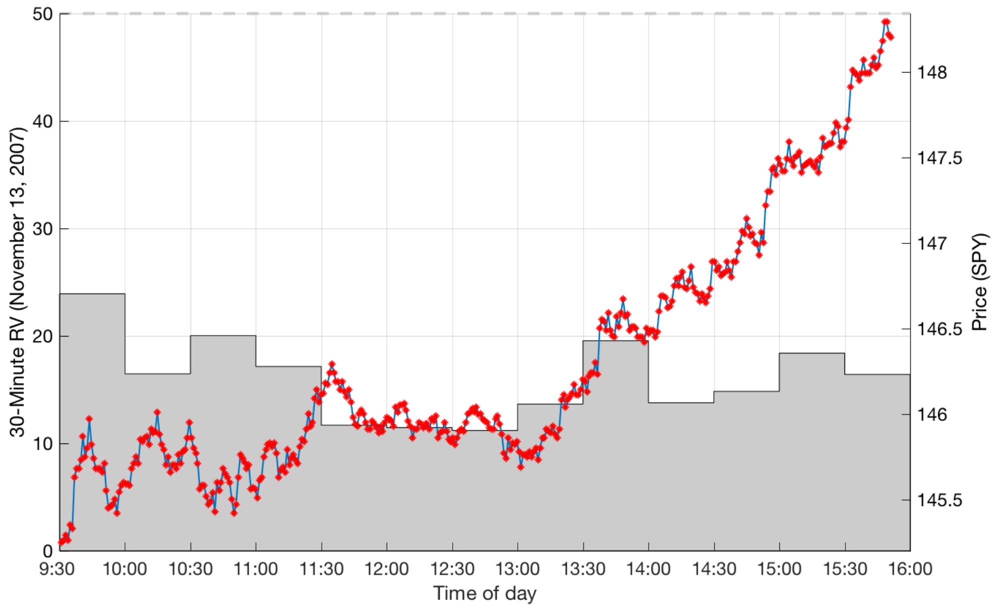

Figure 9: Intraday prices and the realized measure of volatility – November 13, 2007

Tuesday, December 11, 2007 (+83%)

On December 11, 2007, the S&P 500 index fell by 2.5%, while the Dow Jones Industrial Average lost 294 points, or 2.1%, and Nasdaq lost 2.5%. The markets were relatively calm in the morning and then up until about 14:15, when they suddenly went into a tailspin while volatility jumped from about 10% to 70% (at an annualized rate). The main news stories of the day were related to the FOMC meeting that resulted in a 25 b.p. reduction of the Fed Funds Rate to 4.25%, which was announced at 14:15. Other news that morning included the CEO of Freddie Mac, Richard Syron, announcing that Freddie Mac would loose an additional $5.5 billion to $7.5 billion on top of the $4.5 billion losses projected previously.

From Figure 10 it is evident that the FOMC announcement triggered the falling prices in the afternoon. The market had expected reduction of the FFR by 50 b.p. and the surprise had an instant market impact that increased volatility for the remainder of the day, see Birru and Figlewski (2010).

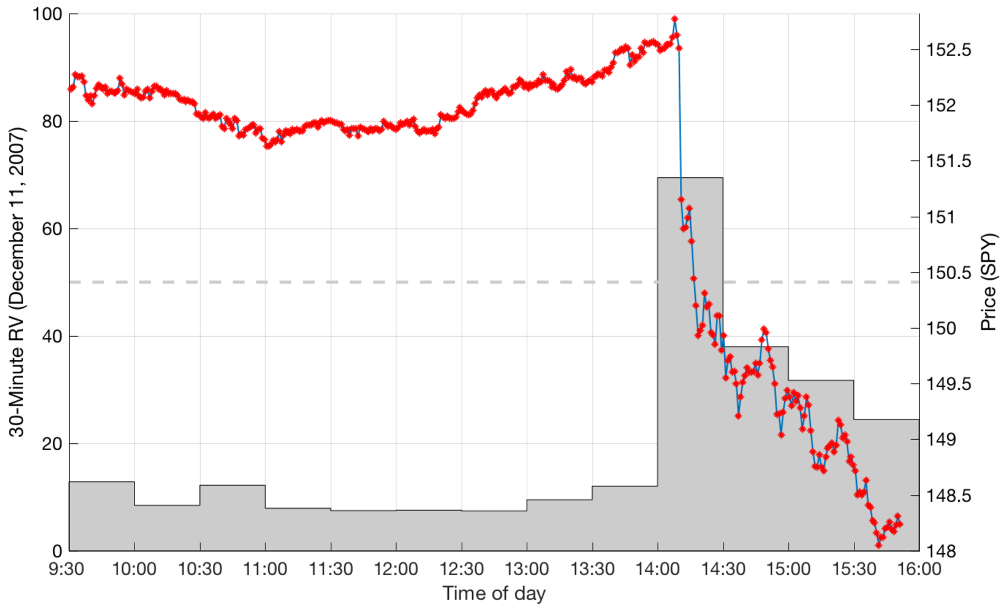

Figure 10: Intraday prices and the realized measure of volatility – December 11, 2007

Friday, December 21, 2007 (-20%)

Stocks rose early on December 21, 2007 until the announcement that Merrill Lynch, which was deeply affected by the credit crisis, was in negotiations with Temasek Holdings (a Singapore's state investment firm) to sell a part of Merrill Lynch. In addition, the Wall Street Journal reported impressive earnings from BlackBerry maker Research in Motion. As a consequence, the Dow Jones Industrial Average had gained about 1.2% during the first hour of trading, S&P 500 index gained 1.3%, and Nasdaq climbed about 1.3%.

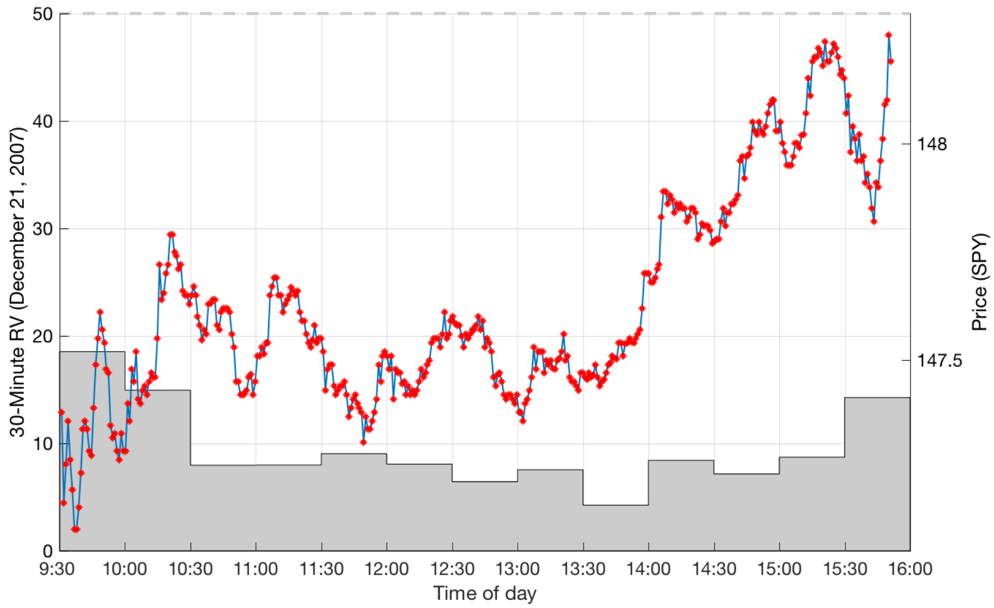

Figure 11: Intraday prices and the realized measure of volatility – December 21, 2007

In the afternoon on December 21, 2007 it was announced that the plans for a Super SIV (structured investment vehicle) were abandoned. The announcement was followed by the statement that "it is not needed at this time", which the markets may have viewed as good news. The Super SIV, formally named Master Liquidity Enhancement Conduit, was intended to resolve liquidity problems that would otherwise cause fire sales of the SIVs assets. Short term financing was increasingly becoming difficult due to market concerns over the SIVs exposure to subprime mortgages. The consortium behind the Super SIV included major financial institutions, including Citigroup, JPMorgan Chase, Bank of America, Wachovia, and Fidelity. The Super SIV was backed by the Treasury Department but critics, including former Federal Reserve chief, Alan Greenspan, claimed that the Super SIV was a bailout of banks, and that it would do more harm than good.

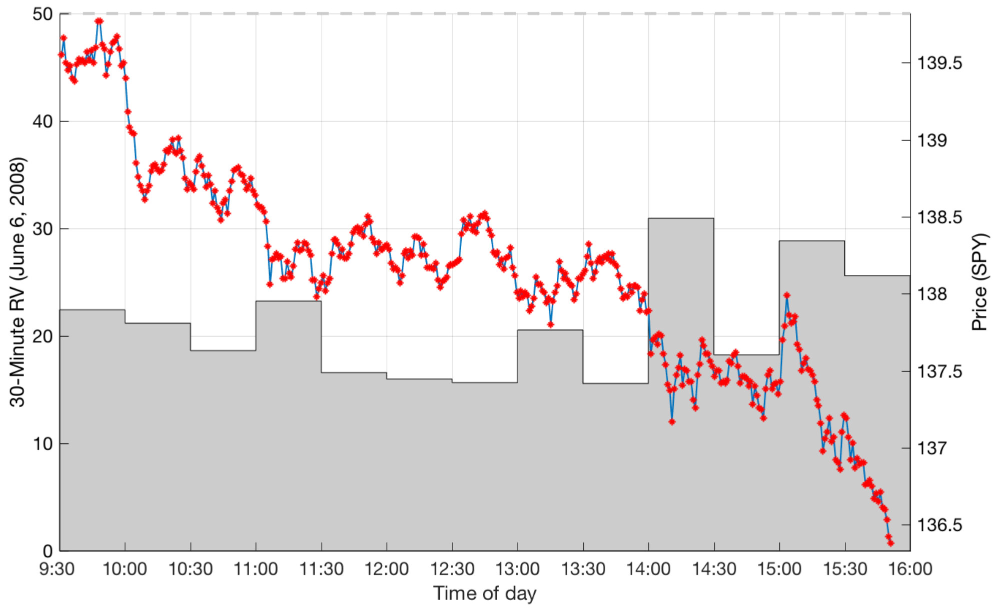

Figure 12: Intraday prices and the realized measure of volatility – June 6, 2008

Friday, June 6, 2008 (+51%)

Early in the morning, Dow, Nasdaq and S&P were down after the May jobs report announced the biggest surge in unemployment since 1986. The unemployment rate increased to 5.5% from 5.0% in April, greatly exceeding the expected rise to 5.1%. The jobs report came on the same day that oil prices jumped to $134 as the dollar lost value against the euro and the yen. It also comes the day after S&P decided to cut the AAA rating of the two largest bond insurers, MBIA (the world's largest bond insurer) and Ambac (the second largest insurer). Moreover, S&P warned that additional downgrades were possible, in anticipation of further losses from mortgage backed securities. MBIA and Ambac ratings were downgraded two notches from AAA to AA, which led to stricter capital requirement.

On that day, the Dow Jones Industrial Average lost 395 points, or 3.1%, its biggest one day decline since the start of the subprime mortgage crisis (February, 2007).

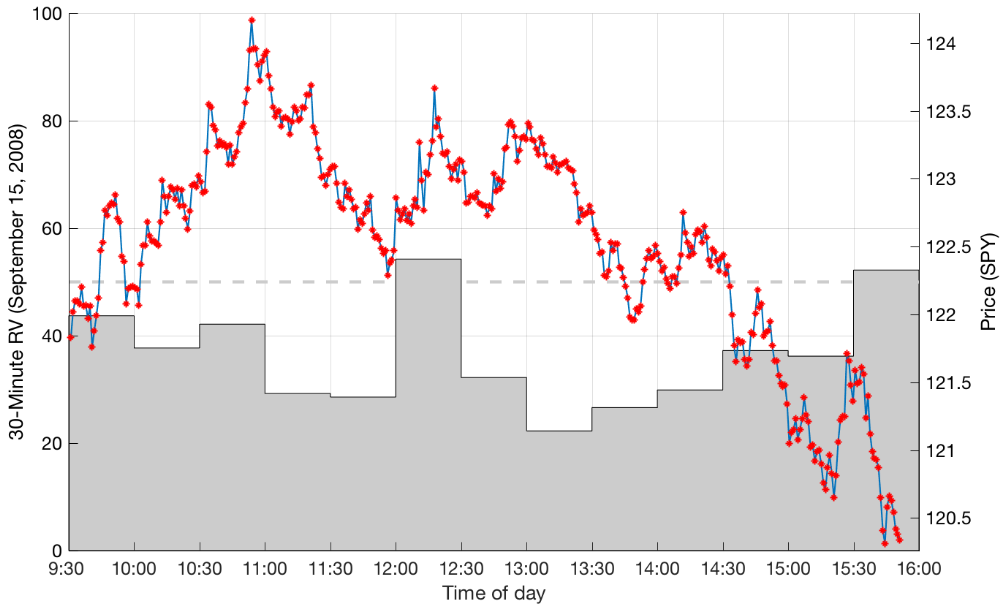

Figure 13: Intraday prices and the realized measure of volatility — September 15, 2008

Monday, September 15, 2008 (+50%)

On September 15, 2008 the Dow Jones Industrial Average index fell by 504.49 points (-4.4%), which was the largest decline since 9/11. The day followed the weekend where Lehman Brothers filed for bankruptcy protection, which was the largest bankruptcy proceeding in the United States history. The collapse of Lehman Brothers made the severity of the crisis crystal clear, and reinforced the concerns that the crisis was systemic and would spread throughout the financial sector and beyond. Merrill Lynch was also severely distressed, but did not file for bankruptcy because Bank of America agreed to purchase Merrill Lynch for $50 billion in stock.

In an attempt to counter these events, the Federal Reserve doubled the size of its Term Securities Lending Facility (TSLF) program to $200 billion and widened the asset group eligible as collateral for Treasury loans. In an attempt to dampen the extent to which the financial turmoil would spread to Europe, the European Central Bank and Bank of England injected €30 billion and £5 billion of capital, respectively.

Monday, September 29, 2008 (+95%)

The second largest volatility shock occurred on September 29, 2008. As shown in Figure 14, prices plunged significantly in the afternoon between 1:30 pm and 1:45 pm. At that time, the House of Representatives rejected (with a 228-205 vote) the Emergency Economic Stabilization

Act of 2008, which triggered a tailspin in the stock market. The banking rescue package was to authorize the Treasure to spend up to $700 billion for purchasing toxic assets, mainly mortgage-backed securities, and supply cash directly to banks. By the end of the day, the Dow had fallen by 777 points – the largest drop in the history – while the S&P 500 index was down by 8.8% - its largest percentage drop since the crash of '87.

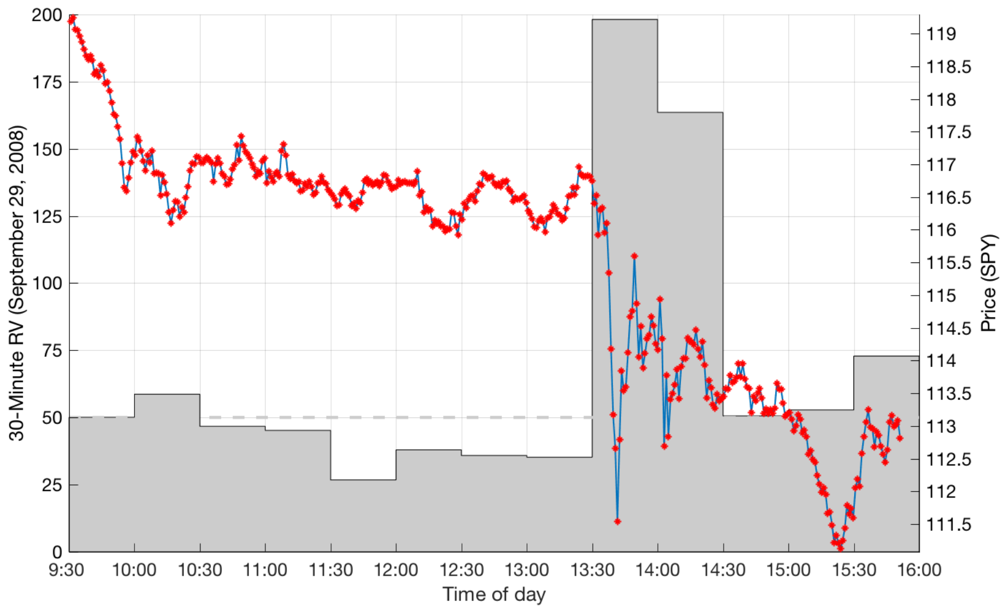

Figure 14: Intraday prices and the realized measure of volatility — September 29, 2008

There was other news on September 29, 2008, that may have contributed to the market turmoil, albeit to a lesser extent. Wachovia announced it was selling its banking operation to Citigroup, and while Wachovia shares lost 81% of their value in the afternoon, Citigroup lost about 12%. The British government nationalized the mortgage lender Bradford & Bingley PLC and some European banks collapsed. The German commercial property lender Hypo Real Estate Group made use of a government-facilitated credit line, due to difficulties in the international credit market. The government of Iceland took control of Glitnir, the country's third largest bank, to prevent its collapse. Moreover, over the weekend, Fortis was partially nationalized, receiving € 11.2 billion capital injection from the Netherlands, Belgium, and Luxembourg.

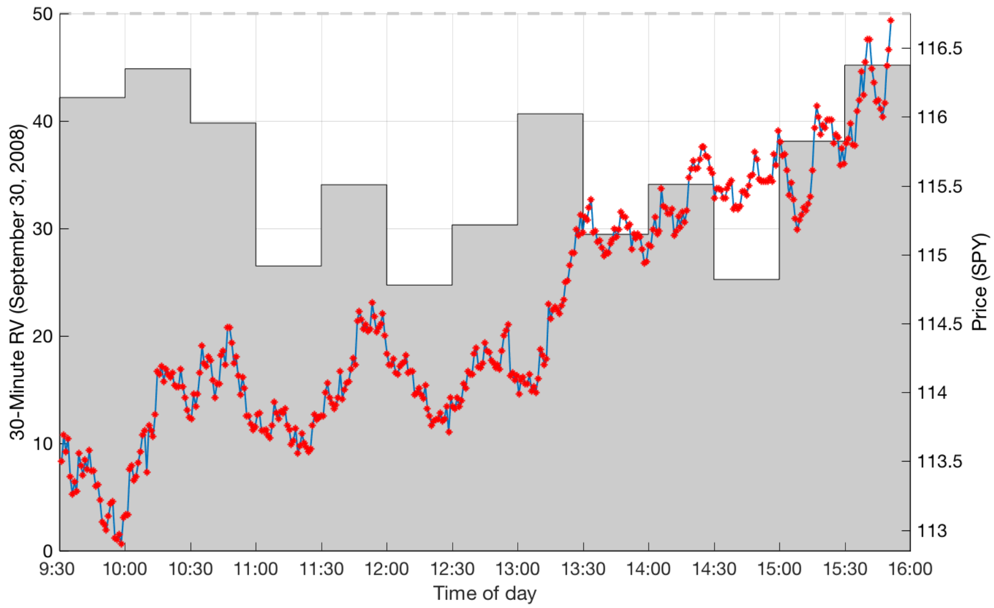

Figure 15: Intraday prices and the realized measure of volatility – September 30, 2008

Tuesday, September 30, 2008 (-20%)

Stock prices rebounded the day after the Congress failed to pass the government's $700 billion rescue plan. The DJIA increased by 485 points that partially reversed the 777 points decline on the previous day. The Standard & Poor's 500 index and the Nasdaq composite both gained about 5%. Most of the rebound occurred late in the day after the Federal Deposit Insurance Corporation announced an enhanced deposit insurance with increased limits, a move that was supported by both presidential candidates, Barack Obama and John McCain.

Monday, October 13, 2008 (-22%)

Stock markets around the world rallied the day in response to several new policies introduced by the US and European Government. The US stock markets increased, after the European markets increased earlier in the day: London's FTSE 100 was up 4.9%, the CAC 40 in Paris was up 6.9%, and the DAX in Frankfurt was up 8.0%.

The leaders of 15 European nations gathered in Paris at a first formal meeting, since the launch of the Euro currency in 1999. Their main goal was to adopt measures to combat credit crisis in Europe. The meeting was organized around four panel discussions on the following themes: i) facilitating the access of banks to capital; ii) global plans for governments to rescue banks through direct capital injections (such as buying soured mortgage assets from banks and injections of capital); iii) an efficient recapitalization of distressed banks and other appropriate means to support the banking system; iv) urging regulators to ease the "mark-to-market" accounting requirements based on the evaluation of assets at their current price. There was a general agreement to act together in a comprehensive wide-ranging plan to rescue the troubled banking system by adding capital through investment and by guaranteeing interbank lending.

Shortly before stocks started trading on October 13, 2008, the British Treasury announced the investment of \$63 billion in three major banks, Royal Bank of Scotland, HBOS, and Lloyds TSB. Other positive news included an unprecedented move by the Federal Reserve Bank, which announced that an unlimited amount of dollars would be available to the central banks: Bank of England, European Central Bank, and the Swiss National Bank.

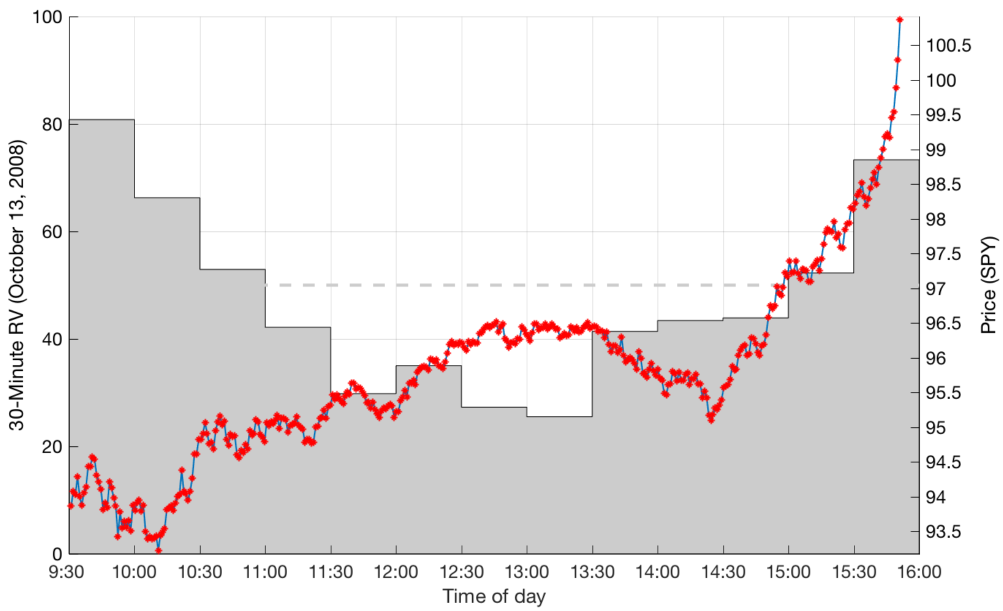

Figure 16: Intraday prices and the realized measure of volatility – October 13, 2008

The French president, Nicolas Sarkozy, committed €360 billion in liquidity to French banks, the German government announced a rescue package worth of \$671 billion and the prime minister of Spain, Jose Luis Rodriguez Zapatero, announced that Spain would provide up to €100 billion of guarantees for new debt issued by commercial banks in 2008. Moreover, in coordination with other eurozone countries, the Dutch government guaranteed interbank lending up to €200 billion. The European Central Bank committed weekly injections of unlimited euro funds at an interest rate of 3.75%.

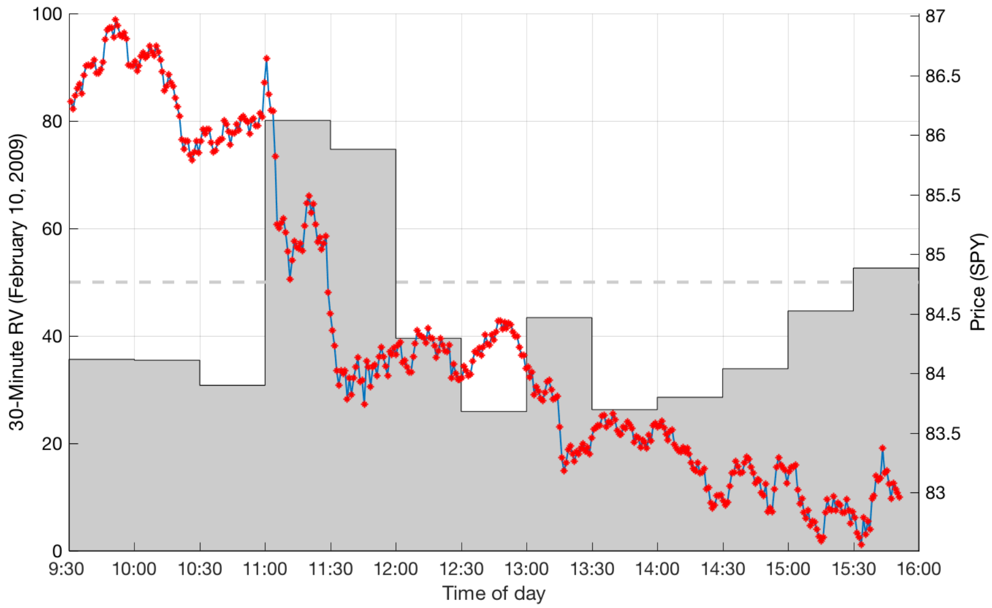

Figure 17: Intraday prices and the realized measure of volatility - February 10, 2009

Tuesday, February 10, 2009 (+54%)

The day began on an optimistic tone in anticipation of the new Financial Stability Plan, that was to replace the original Troubled Asset Relief Program (TARP). The plan was detailed by the US Treasury Secretary, Timothy Geithner, shortly after 11:00 am and had three parts: i) the reinforcement of the stress testing procedures within each banking institution; ii) the development of a new Public-Private Investment Fund, which would provide government capital and government financing helping hence to the recovery of private markets; iii) the revival of the secondary lending markets by a commitment (together with Federal Reserve) up to a a trillion dollars to support a Consumer and Business Lending Initiative.

Nevertheless, the new rescue plan failed to reassure investors, who received it as "a huge disappointment", because it lacked specific details. As a result, the stocks fell during and after Geithner's speech. The Dow Jones Industrial Average lost 382 points (4.6%), which continued in the afternoon. The Standard & Poor's 500 index lost 43 points, or 4.9%. The Nasdaq composite lost 66 points, or 4.2%.

Besides the speech by Geithner, there was bad news from several large companies. General Motors announced it would cut 14% of its workforce around the world, and cut salaries of the remaining employees. Wal-Mart Stores were to layoff 800 workers and UBS 2000 workers, after announcing a $17 billion loss during the last quarter of 2008.

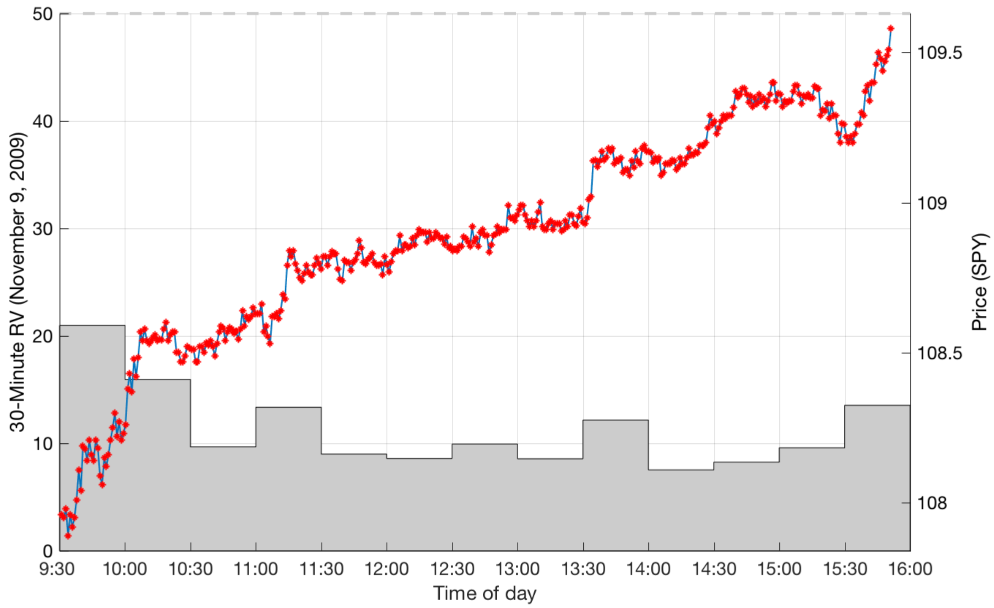

Figure 18: Intraday prices and the realized measure of volatility – November 9, 2009

Monday, November 9, 2009 (-21%)

On November 9, 2009, stock prices rose while volatility fell in response to an announcement made by the *Group of 20* that met over the weekend and confirmed they would keep economic stimulus in place, including the American Recovery and Reinvestment Act of 2009, which is also known as the Obama Stimulus Plan. This economic stimulus plan refers to the $787 billion plan approved by Congress in February, 2009, which was mainly devoted to tax cuts, unemployment benefits, and job creation.

5 Conclusion

In this paper we have analyzed volatility during the recent financial crisis. We made use of high-frequency data in two ways. First, we used high-frequency data to compute realized measures of volatility and the Realized GARCH model for the purpose of determining the days with the largest volatility shocks. Second, having identified the days with the largest volatility shocks, we used intraday high-frequency data to pinpoint the exact timing of these shocks, to the extent this was possible. By comparing specific events and news announcements with the fluctuations observed in the high frequency data, we were in many cases able to identify the main culprits for the volatility shocks. We were also able to rule out events whose timing did not coincide with the shocks.

As an econometric contribution we propose a new variant of the Realized GARCH model, which is sought to be more robust to outliers. The modification is inspired by Harvey (2013), from whom we adopt a simple transformation that dampens the influence of the outliers on the volatility dynamics. The robustified Realized GARCH improves the empirical fit in terms of the log-likelihood function, but the gains are relatively modest, because outliers are rare, and the difference in empirical fit is mainly driven by a few observations. The proposed structure may prove more valuable for time series that are more prone to outliers, or time series for which the the realized measures of volatility are less accurate, that is the case in the present application.

From the estimated model it is straightforward to extract the volatility shock. The volatility shock measures how much the expectation about future volatility changes in response to news on a given day. We proceeded with a detailed analysis of the days with the largest shocks, and used high-frequency data to identify the exact time that some of the shocks occurred, which made it possible to relate to specific events and news stories.

The largest upwards volatility shocks coincided with days with large negative returns, whereas the largest downwards volatility shocks occurred on days with positive returns. The days with large decline in volatility were mainly associated with government interventions. The single largest volatility shock in our sample occurred on February 27, 2007, which was during a relatively calm period with a low level of volatility. This day provides a good example of the benefits of using high-frequency data. There were several major events on February 27, 2007, including a crash on the Chinese stock market and Freddie Mac announcing tighter standards on subprime loans. However, high-frequency data reveal that the volatility shock was mainly caused by a computer glitch in the trading system (just before 3 pm). Without high-frequency data, the relevance of other events might have been overestimated.

A Robustification and Volatility Shock

A.1 Motivating the Robustified Structure

The structure of score-driven models, see Creal et al. (2012, 2013) and Harvey (2013), is motivated by the first order conditions that the true parameter values ought to satisfy. Consider the following example where $y = \sigma z$ with $z \sim t_d$, and $\sigma > 0$ being an unknown scale parameter. If we reparameterize the model with $\lambda = \log \sigma^2$, then the log-likelihood function is

$$\ell(\lambda) = -\tfrac{1}{2}\lambda + c_d - \tfrac{d+1}{2} \log(1 + e^{-\lambda} \tfrac{y^2}{d}),$$

where $c_d = \log[\Gamma(\frac{d+1}{2})/\Gamma(\frac{d+1}{2})/\sqrt{d\pi}]$. The score is therefore

$$s(\lambda) = -\tfrac{1}{2} + \tfrac{d+1}{2} \frac{e^{-\lambda}\frac{y^2}{d}}{1 + e^{-\lambda}\frac{y^2}{d}} = -\frac{1}{2}\left(1 - \frac{\frac{d+1}{d} z^2}{1 + z^2/d}\right) \simeq \frac{1}{2}\left(\tilde{z}^2 - 1\right), \quad \text{with } \tilde{z} = z/\sqrt{1 + z^2/d}.$$

A positive value of $s(\lambda)$ is a signal that the expected log-likelihood may be improved by increasing the value of λ. Similarly, $s(\lambda) < 0$ is an indication that a smaller value of λ may improve the objective. In a time series context, with time varying parameters, $\tilde{z}_t^2 - 1 > 0$ becomes a signal to increase $\lambda_t = \log \sigma_t^2$, whereas $\tilde{z}_t^2 - 1 < 0$ is an indication that λ_t should be lowered. Precisely how much the parameter, λ_t, ought to be changed is less obvious, but a simple starting point is to use a simple autoregressive structure such as $\lambda_t = \omega + \beta\lambda_{t-1} + \alpha s(y_{t-1})$. In the robustified Realized GARCH framework we also want to allow for leverage effects, which is the reason we adopt the specification $\tau(\tilde{z}_t) = \tau_1 \tilde{z}_t + \tau_2(\tilde{z}_t^2 - 1)$. This structure, which includes a linear term, $\tau_1 \tilde{z}_t$, in addition to the score-motivated term, $\tau_2(\tilde{z}_t^2 - 1)$, is identical to that in Hansen et al. (2012) with the exception that $\tilde{z}_t$ has replaced z_t. In our model we maintain the Gaussian distributional specification, and merely use $\tilde{z} = z/\sqrt{1 + z^2/d}$ to reduce the influence of outliers. A fully-fledged DCS/GAS structure is not needed in order to gain the robustness we seek. Adopting t-distributions for z_t and u_t is relatively straightforward, but would be computationally more cumbersome.

A.2 Volatility Shock

The percentage volatility shock to $\sqrt{250h_t}$ is approximately given by $100\left(\exp\frac{v_t}{2}-1\right)$. This follows because

$$\mathbb{E}(\log\sqrt{h_{t+1}}|\mathcal{F}_{t-1})=\tfrac{1}{2}\mathbb{E}(\log h_{t+1}|\mathcal{F}_{t-1})=\tfrac{1}{2}(\omega+\beta\log h_t)=\tfrac{\omega}{2}+\beta\log\sqrt{h_t}\simeq\log\sqrt{h_t},$$

where the last approximation uses that $\omega\simeq 0$ and $\beta\simeq 1$.

From $v_t=\log h_{t+1}-\mathbb{E}(\log h_{t+1}|\mathcal{F}_{t-1})$ it now follows that

$$\exp\tfrac{1}{2}v_t=\exp[\log\sqrt{h_{t+1}}-\mathbb{E}(\log\sqrt{h_{t+1}}|\mathcal{F}_{t-1})]\simeq\frac{\sqrt{h_{t+1}}}{\sqrt{h_t}},$$

which shows that the percentage volatility shock to $\sqrt{h_{t+1}}$ (and hence $\sqrt{250h_{t+1}}$) is approximately given by $100\left(\exp\frac{v_t}{2}-1\right)$.

B Additional Empirical Results

B.1 Estimates from Extended Sample: January 1, 1997 to December 31, 2009.

We have estimated the robust Realized GARCH model for an extended sample period (January 3, 1997 to December 31, 2009). The empirical results for daily SPY close-to-close returns are

$$\begin{aligned}
r_t &= \sqrt{h_t}z_t,\\
\log h_t &= \underset{(0.005)}{0.014}+\underset{(0.004)}{0.967}\log h_{t-1}+\underset{(0.042)}{0.334}\tilde{u}_{t-1}-\underset{(0.009)}{0.151}\tilde{z}_{t-1}+\underset{(0.008)}{0.054}(\tilde{z}_{t-1}^2-1),\\
\log x_t &= \underset{(0.037)}{-0.418}+\underset{(0.049)}{1.038}\log h_t-\underset{(0.009)}{0.133}z_t+\underset{(0.007)}{0.044}(z_t^2-1)+u_t,
\end{aligned}$$

with $\hat{\sigma}_u^2=\underset{(0.006)}{0.1687},\hat{d}_z=32.975,\hat{d}_u=4.771$, which is in agreement with the model estimated with the shorter sample, given the reported standard errors.

B.2 Comparison of Different Robust Specifications

We explored a range of specifications with different degrees of robustness to outliers. All models are submodels of the following model:

$$
\begin{aligned}
r_t &= \mu + \sqrt{h_t} z_t, \\
\log h_t &= \omega + \beta \log h_{t-1} + \tau(\tilde{z}_{1,t-1}) + \gamma \tilde{u}_{t-1}, \quad \tau(z) = \tau_1 z + \tau_2 (z^2 - 1), \\
\log x_t &= \xi + \varphi \log h_t + \delta(\tilde{z}_{2,t}) + u_t, \qquad \delta(z) = \delta_1 z + \delta_2 (z^2 - 1).
\end{aligned}
$$

The structure for each of the models is as follows, where M0 is the Realized GARCH model, M5 is the specification used in the paper, and M6 is the most general specification:

M0: $z_t = \tilde{z}_{1,t} = \tilde{z}_{2,t}$ and $u_t = \tilde{u}_t$

M1: $z_t = \tilde{z}_{1,t} = \tilde{z}_{2,t}$, and $\tilde{u}_t = u_t / \sqrt{1 + (u_t/\sigma_u)^2 / d_u}$

M2: $\tilde{z}_{1,t} = \tilde{z}_{2,t} = \tilde{z}_t$ with $\tilde{z}_t = z_t / \sqrt{1 + z_t^2 / d_z}$ and $u_t = \tilde{u}_t$

M3: $\tilde{z}_{1,t} = z_t / \sqrt{1 + z_t^2 / d_{1z}}$, $\tilde{z}_{2,t} = z_t / \sqrt{1 + z_t^2 / d_{2z}}$, and $u_t = \tilde{u}_t$

M4: $z_t = z_{2,t}$, $\tilde{z}_{1,t} = z_t / \sqrt{1 + z_t^2 / d_{1z}}$ and $u_t = \tilde{u}_t$

M5: $z_t = z_{2,t}$, $\tilde{z}_{1,t} = z_t / \sqrt{1 + z_t^2 / d_{1z}}$ and $\tilde{u}_t = u_t / \sqrt{1 + (u_t/\sigma_u)^2 / d_u}$

M6: $\tilde{z}_{1,t} = z_t / \sqrt{1 + z_t^2 / d_{1z}}$ $\tilde{z}_{2,t} = z_t / \sqrt{1 + z_t^2 / d_{2z}}$, and $\tilde{u}_t = u_t / \sqrt{1 + (u_t/\sigma_u)^2 / d_u}$

The empirical results are presented in Table B.1. As previously noted, the robustified Realized GARCH model controls the impact of jumps on volatility and on the realized measure. This can be done in a variety of ways, and each of the seven models has a degree of robustness. M6 has the most flexible specification and M0 is the original specification without robustness.

In this section we shed light on the robustified Realized GARCH structure (both general and simplified forms) and subsequently compared its performances in terms of empirical fit with those of the standard Realized GARCH. To this end, we estimate the various specifications with robustness (M1-M6) and compare them to the standard Realized GARCH model (M0). The empirical results for the sample period 2006 to 2009 are presented in Table B.1.

The highest value of the log-likelihood is achieved by the most general specification, M6, albeit it is closely followed by M5, and the difference between these two models is not statistically significant. Moreover, the new parameter of the transformed innovation term that appears into the measurement equation of M6 is quite large, which suggests that $\tilde{z}_{2,t} = z_t$ might be reasonable. By adopting the model M5, we are only introducing robustness to outliers in the GARCH equation, while leaving the measurement equation unchanged. The estimated parameter associated with the number of degrees of freedom appearing in the transformed innovation term $\tilde{u}_t$ ($d_u = 5.904$) is lower than that associated with $\tilde{z}_{1,t}$ ($d_{1,z} = 27.689$), which suggests that the influence of the outliers related to the realized volatility series requires the highest extent of dampening. The log-likelihood for M5 is six units greater than the classical Realized GARCH specification, which indicates a valuable statistical benefit of incorporating robustness in the GARCH equation.

Table B.1: Parameter estimates for each of the seven model specifications: The Realized GARCH model (M0) and the six robustified models

	M0 Realized GARCH	**M1**	**M2**	**M3**	**M4**	**M5** (Preferred)	**M6**
d_{1z}			63.531	29.487	33.359	27.689	24.792
d_{2z}			63.531	290.770			290.787
d_u		12.768				5.904	5.895
h_0	0.797	0.812	0.782	0.797	0.803	0.824	0.819
ω	0.006	0.007	0.010	0.015	0.014	0.017	0.018
β	0.972	0.972	0.971	0.968	0.968	0.969	0.969
γ	0.368	0.402	0.364	0.354	0.351	0.407	0.411
τ_1	-0.171	-0.171	-0.177	-0.180	-0.178	-0.180	-0.182
τ_2	0.025	0.025	0.043	0.056	0.053	0.056	0.059
ξ	-0.518	-0.519	-0.516	-0.528	-0.531	-0.532	-0.529
φ	1.006	1.005	0.994	1.014	1.022	1.019	1.012
δ_1	-0.128	-0.129	-0.133	-0.130	-0.129	-0.132	-0.133
δ_2	0.037	0.036	0.052	0.042	0.038	0.036	0.040
σ_u^2	0.157	0.157	0.156	0.154	0.154	0.154	0.154
AIC	4026.1	4026.3	4024.9	4020.1	4019.2	**4017.4**	4018.3
BIC	4080.0	4085.1	4088.6	4083.8	**4078.0**	4081.1	4086.9
-logL	2002.1	2001.2	1999.5	1997.0	1997.6	1995.7	**1995.1**

Table B.2 reports the values of the ten largest positive volatility shocks along with the corresponding dates of occurrence, for each of the seven estimated models. The models are largely in agreement about the dates on which the largest volatility shocks occurred on, but the estimated magnitude of the volatility shocks differs. The dampening effect of outliers are evident from the estimated value of v_t, but effectively, only the three largest volatility shocks are substantially smaller for the robustified specifications.

Table B.2: Ten largest positive volatility shocks for each of the seven specifications

M0		M1		M2		M3		M4		M5		M6	
Date	v_t	**Date**	v_t	**Date**	v_t	**Date**	v_t	**Date**	v_t	**Date**	v_t	**Date**	v_t
20070227	2,295	20070227	2,298	20070227	2,200	20070227	1,670	20070227	1,654	20070227	1,558	20070227	1,576
20080929	1,314	20080929	1,304	20080929	1,391	20080929	1,373	20080929	1,375	20080929	1,334	20080929	1,331
20071211	1,213	20071211	1,192	20071211	1,277	20071211	1,260	20071211	1,256	20071211	1,208	20071211	1,214
20080606	0,791	20080606	0,785	20080606	0,845	20090210	0,871	20090210	0,868	20090210	0,866	20090210	0,869
20090210	0,779	20090210	0,779	20090210	0,830	20080606	0,842	20080606	0,842	20080606	0,827	20080606	0,825
20070726	0,731	20070726	0,712	20080915	0,763	20080915	0,802	20080915	0,800	20080915	0,808	20080915	0,808
20080915	0,705	20070710	0,712	20070726	0,756	20070710	0,777	20070710	0,768	20070710	0,785	20070710	0,794
20070710	0,701	20080915	0,707	20070710	0,750	20070726	0,767	20070726	0,761	20070313	0,760	20070313	0,765
20070313	0,662	20070313	0,666	20070313	0,720	20070313	0,754	20070313	0,748	20071101	0,729	20070726	0,734
20071101	0,638	20071101	0,647	20071101	0,685	20071101	0,710	20071101	0,706	20070726	0,728	20071101	0,733

Note: This table presents the ten largest positive volatility shocks computed as $v_t = \tau(\tilde{z}_t) + \gamma\tilde{u}_t$, along with the corresponding dates of occurrence.

References

Andersen, T. G., Bollerslev, T., 1998. Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. International Economic Review 39 (4), 885–905.

Andersen, T. G., Bollerslev, T., Diebold, F. X., Labys, P., 2001. The distribution of realized exchange rate volatility. Journal of the American Statistical Association 96, 42–55.

Andersen, T. G., Bollerslev, T., Diebold, F. X., Labys, P., 2003. Modeling and forecasting realized volatility. Econometrica 71, 579–625.

Banulescu, D., Hansen, P. R., Huang, Z., Matei, M., 2017. Web appendix for ”volatility during the financial crisis through the lens of high frequency data: A Realized GARCH approach”. https://sites.google.com/site/peterreinhardhansen/.

Barndorff-Nielsen, O. E., Hansen, P. R., Lunde, A., Shephard, N., 2008. Designing realised kernels to measure the ex-post variation of equity prices in the presence of noise. Econometrica 76, 1481–536.

Barndorff-Nielsen, O. E., Hansen, P. R., Lunde, A., Shephard, N., 2009. Realised kernels in practice: Trades and quotes. Econometrics Journal 12, 1–33.

Barndorff-Nielsen, O. E., Hansen, P. R., Lunde, A., Shephard, N., 2011. Multivariate realised kernels: consistent positive semi-definite estimators of the covariation of equity prices with noise and non-synchronous trading. Jounal of Econometrics 162, 149–169.

Barndorff-Nielsen, O. E., Shephard, N., 2002. Econometric analysis of realised volatility and its use in estimating stochastic volatility models. Journal of the Royal Statistical Society **B** 64, 253–280.

Barndorff-Nielsen, O. E., Shephard, N., 2007. Advances in Economics and Econometrics. Theory and Applications, Ninth World Congress. Econometric Society Monographs. Cambridge University Press, Ch. Variation, jumps and high frequency data in financial econometrics, pp. 328–372.

Birru, J., Figlewski, S., 2010. The impact of the Federal Reserve's interest rate target announcement on stock prices: A closer look at how the market impounds new information. Working paper.

Bollerslev, T., Wooldridge, J. M., 1992. Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariance. Econometric Reviews 11, 143–172.

Creal, D., Koopman, S. J., Lucas, A., 2012. A dynamic multivariate heavy-tailed model for time-varying volatilities and correlations. Journal of Business & Economic Statistics 29, 552–563.

Creal, D. D., Koopman, S. J., Lucas, A., 2013. Generalized autoregressive score models with applications. Journal of Applied Econometrics 28, 777–795.

Engle, R. F., 2002. New frontiers of ARCH models. Journal of Applied Econometrics 17, 425–446.

Engle, R. F., Gallo, G., 2006. A multiple indicators model for volatility using intra-daily data. Journal of Econometrics 131, 3–27.

Engle, R. F., Ng, V., 1993. Measuring and testing the impact of news on volatility. Journal of Finance 48, 1747–1778.

Hansen, P. R., Horel, G., 2009. Quadratic variation by Markov chains. Working paper.

Hansen, P. R., Huang, Z., 2016. Exponential GARCH modeling with realized measures of volatility. Journal of Business & Economic Statistics 34 (2), 269–287.

Hansen, P. R., Huang, Z., Shek, H., 2012. Realized GARCH: A joint model of returns and realized measures of volatility. Journal of Applied Econometrics 27, 877–906.

Hansen, P. R., Lunde, A., Voev, V., 2014. Realized beta GARCH: A multivariate GARCH model with realized measures of volatility. Journal of Applied Econometrics 29, 774–799.

Harvey, A. C., 2013. Dynamic models for volatility and heavy tails: With applications to financial and economic time series. Cambridge University Press.

Meddahi, N., 2002. A theoretical comparison between integrated and realized volatility. Journal of Applied Econometrics 17, 479–508.

Mykland, P. A., Zhang, L., 2009. Inference for continuous semimartingales observed at high frequency: A general approach. Econometrica 77, 1403–1445.

Shephard, N., Sheppard, K., 2010. Realising the future: Forecasting with high frequency based volatility (HEAVY) models. Journal of Applied Econometrics 25, 197–231.

Visser, M. P., 2011. GARCH parameter estimation using high-frequency data. Journal of Financial Econometrics 9, 162–197.

Zhang, L., Mykland, P. A., Aït-Sahalia, Y., 2005. A tale of two time scales: Determining integrated volatility with noisy high frequency data. Journal of the American Statistical Association 100, 1394–1411.